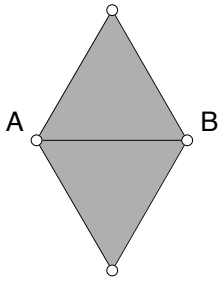
Tessellating with Regular Polygons

Name(s): _____

You've probably seen a floor tiled with square tiles. Squares make good tiles because they can cover a surface without any gaps or overlapping. This kind of tiling is sometimes called a *tessellation*. Are there other shapes that would make good tiles? In this investigation, you'll discover which regular polygons tessellate. You'll need custom tools for creating equilateral triangles, squares, regular pentagons, regular hexagons, and regular octagons. Each custom tool must create its figure from the endpoints of one side of the polygon—not from the center. Such custom tools come with the sketch below, but you may want to create them yourself.

Sketch and Investigate

1. Open the sketch **Islamic_Tessellations.gsp** if it is not already opened.



Click on the Custom tools icon (the bottom tool in the Toolbox) and choose the desired tool from the menu that appears. Click twice in the sketch to use the tool.

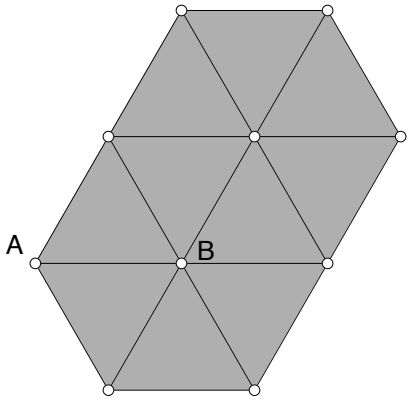
2. On a blank page, or in a blank sketch, use a custom tool to construct an equilateral triangle. Pay attention to the direction in which the triangle is created as you use the tool.

3. Use the custom tool again, this time clicking in the opposite direction, to construct a second equilateral triangle attached to the first.

If your triangles don't stay attached, undo until the second triangle goes away, then try again. You must always start and end your dragging with the cursor positioned on an existing point.

4. Drag several points on the triangles to make sure they're really attached.

5. Keep attaching triangles to edges of existing triangles until you have triangles completely surrounding at least two points.



Q1 So far, you've demonstrated that equilateral triangles can tessellate. You can tile the plane with them without gaps or overlapping. Why do equilateral triangles work? (*Hint:* It has to do with their angles.)

6. Repeat the investigation with squares, regular pentagons, regular hexagons, and regular octagons.

Q2 Which of the regular polygons you tried will tessellate and which won't? Why? Answer on a separate sheet.

Q3 Do you think a regular heptagon (seven sides) would tessellate? Explain.

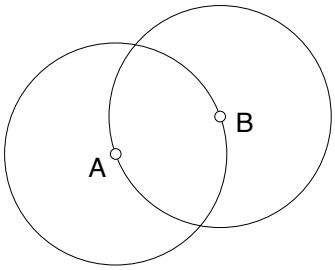
Daisy Designs

Name(s): _____

A daisy design is a simple design that you can create using only a compass. From the basic daisy, you can create more complex designs based on the regular hexagon.

If the point labels don't appear, don't worry. They'll help you follow the directions, but you don't actually want them in your design.

1. Construct circle *AB* (a circle with center *A* and radius point *B*).



2. Construct circle *BA*. Be sure you start your circle with the cursor positioned at point *B* and that you finish your circle with the cursor positioned at point *A*.

If both circles are not controlled by just points *A* and *B*, undo and try again.

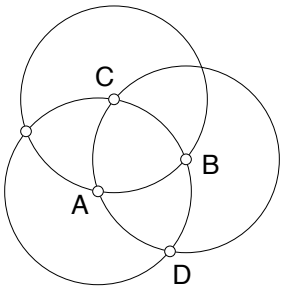
3. Drag point *A* and point *B* to confirm that both circles are controlled by these two points.

4. Construct point *C* and point *D,* the two points of intersection of these circles.

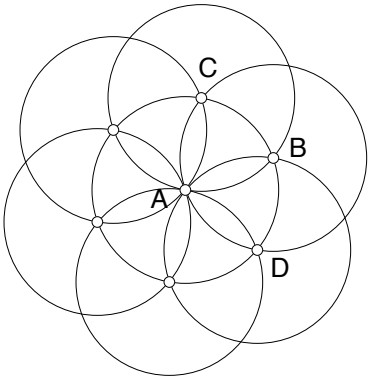
Be sure all the circles are connected by constructing them from intersections to existing points in the sketch. Your final daisy should have exactly seven points.

5. Construct a circle from point *C* to point *A*.

6. Continue constructing circles from new intersection points to point *A*. All these circles should have equal radii. The last circle you construct should be centered at point *D*. When you're done, your sketch should look like the figure below right. You should be able to drag it without making it fall apart.



Step 5

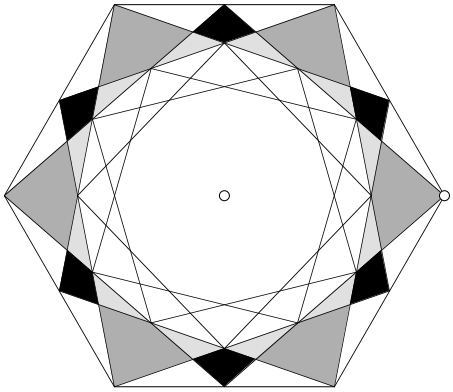


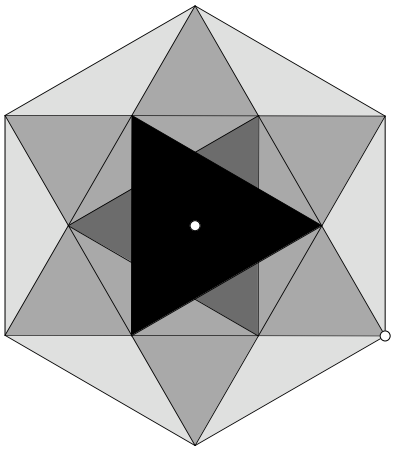
Step 6

7. The six points of your daisy (besides point *A*) define six vertices of a regular hexagon. Use the **Segment** tool to connect the tips of the petals your design to form a regular hexagon; then drag point *B* and observe the way your design changes.

You could construct polygon interiors and experiment with color. You could also construct arcs (select a circle and two points on it) and arc sector and arc segment interiors (select an arc). However, you can

probably get better results by printing out the basic line design and adding color and shading by hand. Once you have all the lines and polygon interiors you want, you can hide unneeded points. Don't hide your original two points, though, because you can use these points to manipulate your figure.





For tips on making and using custom tools, choose **Toolbox** from the Help menu, then click on the Custom Tools link.

Explore More

1. Use the daisy construction to create a custom tool for a regular hexagon. Save the new tool in the Tool Folder (next to the application itself on your hard drive) so it can be used in any open sketch.

A Tumbling-Block Design

Name(s): _____

A tumbling-block design is commonly found in Islamic tilings. We can call it an example of "op art" because of its interesting optical effect, which is suggested by its name. A tumbling-block design can be created efficiently with Sketchpad using *translations*.

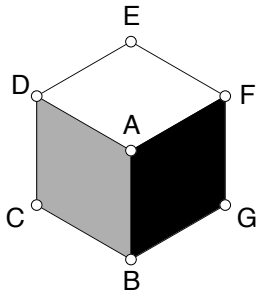
Sketch and Investigate

Use your own method or a custom tool like 6/Hexagon (Inscribed) from the sketch.

1. Construct a regular hexagon.
2. Delete the polygon's interior, if necessary.
3. If the center of the hexagon doesn't already exist, construct it.

To construct a polygon interior, select the vertices in consecutive order; then, in the Construct menu, choose Quadrilateral Interior. Color is in the Display menu.

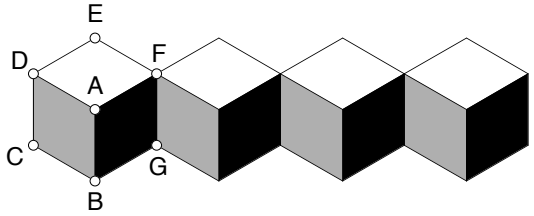
4. Construct segments and polygon interiors, and color regions of the hexagon as shown at right.



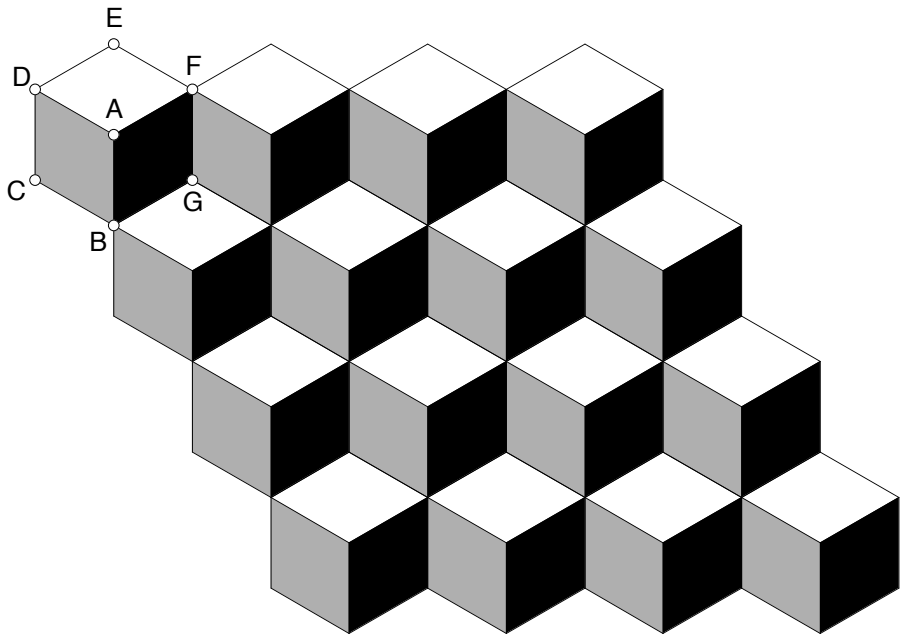
In the next few steps, you'll translate this figure to create a row of blocks.

Select, in order, point *D* and point *F*; then, in the Transform menu, choose Mark Vector.

5. Mark the vector *DF*.
6. Select the nine segments and two interiors (in other words, everything but the points).
7. Translate the selection.
8. Translate again two more times so that you have a row of four blocks.
9. Mark *EG* as a vector and translate the entire row of blocks (except the points) by this vector to create a second row of blocks.



10. Translate again two more times so that you have four rows of blocks.
11. Drag points on your original hexagon to scale and turn the design.
12. Experiment with different color patterns to enhance your design.



Q1 Describe different shapes in this design. What's the shape of the smallest pieces quilters use in this design?

Q2 Describe the optical effect of the design.

Q3 Describe any places where you have seen this design before.

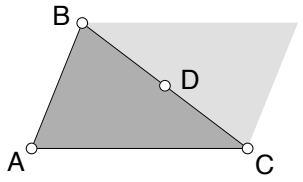
Tessellating with Triangles

Name(s): _____

In this investigation, you'll learn a method for tessellating with any triangle. You'll also discover why all triangles tessellate.

Sketch and Investigate

1. In the lower left corner of your screen, construct triangle *ABC*.



Select the three vertices; then, in the Construct menu, choose Triangle Interior.

2. Construct its interior.

3. Construct midpoint *D* of side *BC*.

Double-click the point to mark it as a center. Select the triangle interior; then, in the Transform menu, choose Rotate.

4. Mark point *D* as a center and rotate the triangle interior by 180°.

5. Give the rotated image a different color.

Color is in the Display menu.

Q1 Drag points and observe the shape formed by the two triangles (the original together with its rotated image). What shape is this? _____

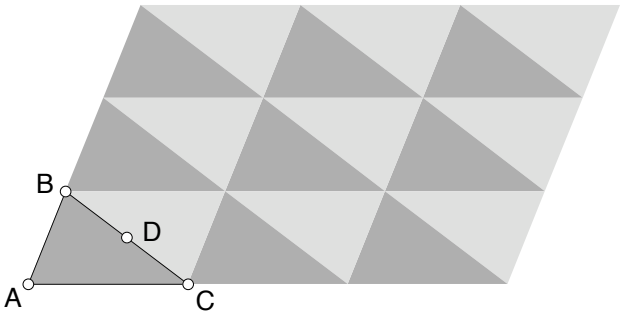
Select, in order, point *A* and point *C*; then, in the Transform menu, choose Mark Vector. A brief animation indicates the mark. Select the two interiors; then, in the Transform menu, choose Translate.

6. Mark *AC* as a vector and translate the two interiors by the marked vector. Translate two more times to make a total of eight triangles.

Q2 Drag to confirm that the top and bottom edges of this row of triangles are always straight lines. What does that demonstrate about the sum of the angle measures in the original triangle? Explain. (Write your explanation on a separate sheet.)

7. Mark vector *AB* and translate the entire row by this vector. Repeat until triangles begin to fill your screen.

8. Drag to confirm that no matter what shape your original triangle has, it will always tessellate.



Q3 Look at a point in the tessellation that is completely surrounded by triangles. What is the sum of the angles surrounding this point? Why? (Write your explanation on a separate sheet.)

Tessellations Using Only Translations

Name(s): _____

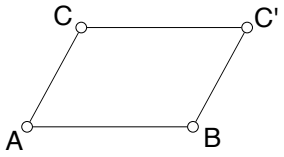
In this activity, you'll learn how to construct an irregularly shaped tile based on a parallelogram. Then you'll use translations to tessellate your screen with this tile.

Sketch

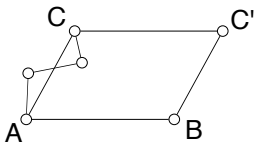
1. Construct s*AB* in the lower-left corner of your sketch, then construct point *C* just above s*AB*.

Select, in order, point *A* and point *B*; then, in the Transform menu, choose Mark Vector. Select point *C*; then, in the Transform menu, choose Translate.

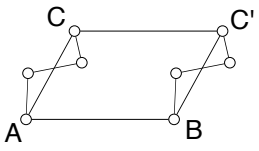
2. Mark the vector from point *A* to point *B* and translate point *C* by this vector.
3. Construct the remaining sides of your parallelogram.



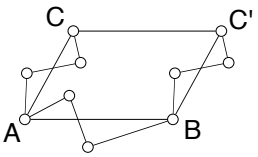
Steps 1–3



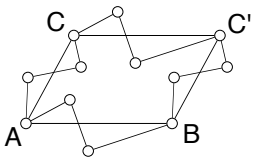
Step 4



Step 5



Step 6

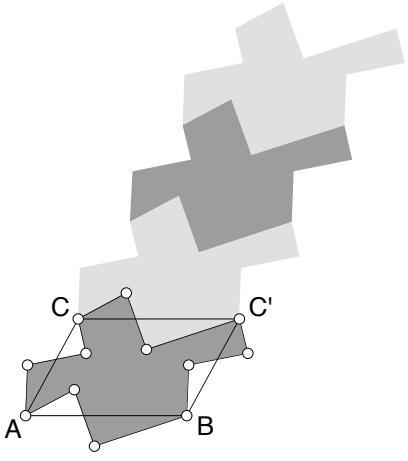


Step 7

4. Construct two or three connected segments from point *A* to point *C*. We'll call this *irregular edge AC*.
5. Select all the segments and points of irregular edge *AC* and translate them by the marked vector. (Vector *AB* should still be marked.)
6. Make an irregular edge from *A* to *B*.
7. Mark the vector from point *A* to point *C* and translate all the parts of irregular edge *AB* by the marked vector.

Select the vertices in consecutive order; then, in the Construct menu, choose Polygon Interior.

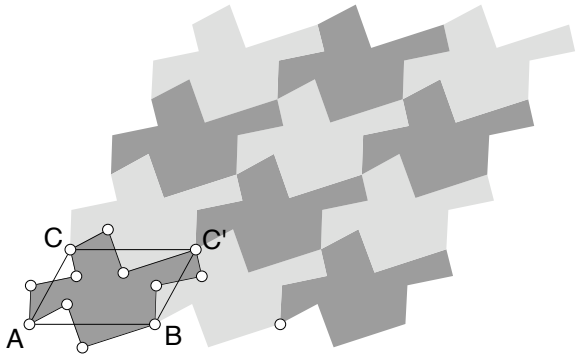
8. Construct the polygon interior of the irregular figure. This is the tile you will translate.
9. Translate the polygon interior by the marked vector. (You probably still have vector *AC* marked.)
10. Repeat this process until you have a column of tiles all the way up your sketch. Change the color of every other tile to create a pattern.



Steps 8–10

11. Mark vector *AB*. Then select all the polygon interiors in your column of tiles and translate them by this marked vector.

12. Continue translating columns of tiles until you fill your screen. Change colors of alternating tiles so you can see your tessellation.



Steps 11 and 12

13. Drag vertices of your original tile until you get a shape that you like or that is recognizable as some interesting form.

Explore More

1. Animate your tessellation. To do this, select the original polygon (or any combination of its vertex points) and choose **Animate** from the Display menu. You can also have your points move along paths you construct. To do this, construct the paths (segments, circles, polygon interiors—anything you can construct a point on) and then merge vertices to paths. (To merge a point to a path, select both and choose **Merge Point to Path** from the Edit menu.) Select the points you wish to animate and, in the Edit menu, choose **Action Buttons | Animation**. Press the Animation button. Adjust the paths so that the animation works in a way you like, then hide the paths.

2. Use Sketchpad to make a translation tessellation that starts with a regular hexagon as the basic shape instead of a parallelogram. (*Hint:* The process is very similar; it just involves a third pair of sides.)

Tessellations That Use Rotations

Name(s): _____

Tessellations that use only translations have tiles that all face in the same direction. Using rotations, you can make a tessellation with tiles facing in different directions. The designs in a rotation tessellation have rotation symmetry about points in the tiling.

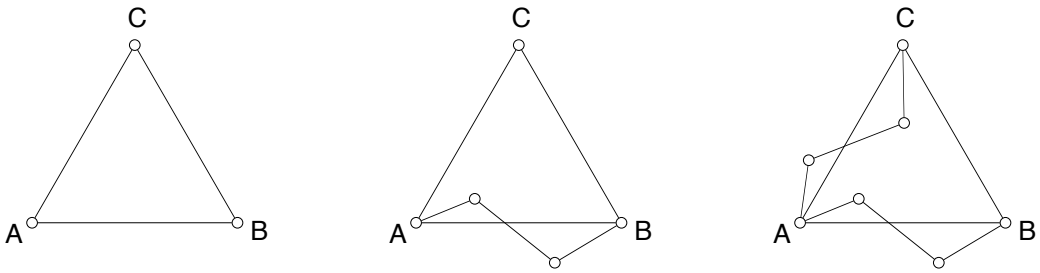
Sketch and Investigate

Use a custom tool (such as one in the sketch Islamic _Tessellations.gsp) or construct the triangle from scratch. If your custom tool constructs an interior, delete the interior.

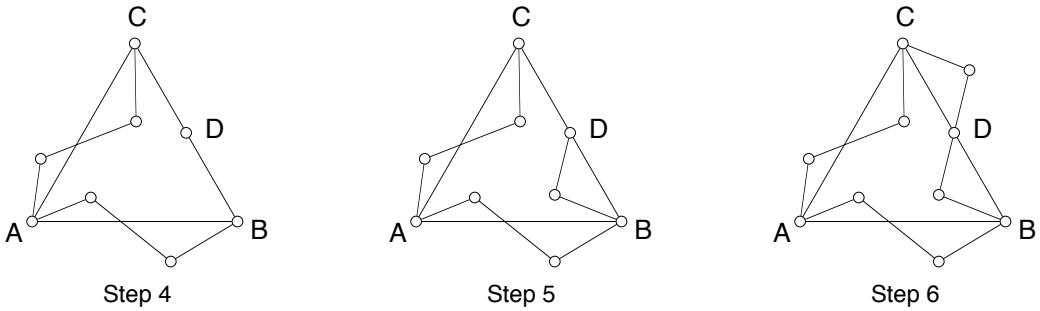
1. Construct equilateral triangle *ABC* as shown below.
2. Construct two or three connected segments from *A* to *B*. We'll call this *irregular edge AB*.

Double-click the point to mark it as a center. Select the segments and points; then, in the Transform menu, choose Rotate.

3. Mark point *A* as a center for rotation. Then rotate all the points and segments of irregular edge *AB* by 60°.

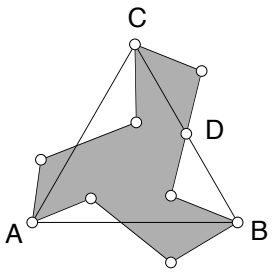


4. Construct midpoint *D* of side *CB*.
5. Construct two connected segments from *B* to *D*. We'll call this *irregular edge BD*.
6. Mark point *D* as a center for rotation. Then rotate the point and segments of irregular edge *BD* by 180°.
7. You have finished the edges of your tile. Drag points to see how they behave. When you're done, make sure none of the irregular edges intersect.



Select the vertices in consecutive order; then, in the Construct menu, choose Polygon Interior.

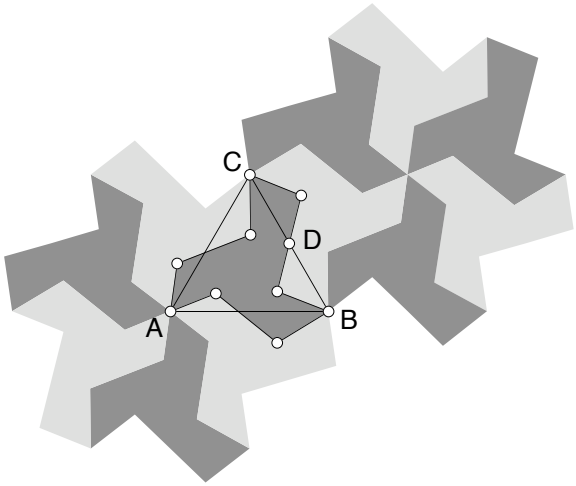
8. Construct the polygon interior with vertices along the irregular edges.



Change the color of your tiles using the Display menu.

9. To begin tessellating, mark point *A* as a center and rotate the tile interior six times by the appropriate number of degrees to surround point *A* with tiles. Change the color of alternate tiles.

10. Mark point *D* as a center and rotate the six tiles by 180°. Reverse their shading as necessary to keep a clear shading pattern.



Q1 Look at the tiles surrounding point *A*. What kind of rotation symmetry would the completed tessellation have about this point?

Q2 Look at the tiles surrounding point *D*. What kind of rotation symmetry would the completed tessellation have about this point?

Q3 Look at the tiles surrounding points *B* and *C* so far. What angle of rotation will you have to use to fill in tiles around these points?

11. Use the appropriate rotations to fill in tiles around points *B* and *C*. If you choose an angle that doesn't work right, undo and try a different angle. Change your answer to Q3, if necessary.

12. Drag vertices of your original tile until you get a shape that you like or that is recognizable as some interesting form.